

AXIAL GAUGE IN QUANTUM SUPERGRAVITY

Giampiero Esposito^{1,2} and Alexander Yu. Kamenshchik³

¹*Istituto Nazionale di Fisica Nucleare, Sezione di Napoli,
Mostra d'Oltremare Padiglione 20, 80125 Napoli, Italy;*

²*Dipartimento di Scienze Fisiche,
Mostra d'Oltremare Padiglione 19, 80125 Napoli, Italy;*

³*Nuclear Safety Institute, Russian Academy of Sciences,
52 Bolshaya Tuskaya, Moscow 113191, Russia.*

Abstract. This paper studies the role of the axial gauge in the semiclassical analysis of simple supergravity about the Euclidean four-ball, when non-local boundary conditions of the spectral type are imposed on gravitino perturbations at the bounding three-sphere. Metric perturbations are instead subject to boundary conditions completely invariant under infinitesimal diffeomorphisms. It is shown that the axial gauge leads to a non-trivial cancellation of ghost-modes contributions to the one-loop divergence. The analysis, which is based on zeta-function regularization, provides a full $\zeta(0)$ value which coincides with the one obtained from transverse-traceless perturbations for gravitons and gravitinos. The resulting one-loop divergence does not vanish. This property seems to imply that simple supergravity is not even one-loop finite in the presence of boundaries.

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1. Introduction

When supergravity theories were introduced and the formalism for their quantization was developed [1–3], there was the hope that the supersymmetry relating bosonic and fermionic fields would have improved the finiteness properties of pure gravity [4–6] (although these theories are *not* perturbatively renormalizable). However, the analysis of ultraviolet divergences is technically so difficult that not even one-loop divergences were completely analyzed in non-trivial backgrounds. More precisely, we are here interested in the semi-classical analysis of quantum supergravity in Riemannian four-manifolds with boundary. Boundary effects are indeed crucial in the path-integral approach to quantum gravity and quantum cosmology [7,8], and the recent progress in the quantization programme of field theories in the presence of boundaries has shed new light on the problems of Euclidean quantum gravity [9–12] and on some properties of Euclidean Maxwell theory [13–16].

In the absence of boundaries, the massless gravitinos of simple supergravity, with unrestricted gauge freedom, can only be studied in Ricci-flat backgrounds [17]. In the presence of boundaries, however, one has to impose boundary conditions which involve both the unprimed and the primed part of the gravitino potential (in two-component spinor notation), hereafter denoted by ψ_μ^A and $\tilde{\psi}_\mu^{A'}$ respectively. Each of these spinor-valued one-forms admits a local description in terms of a second potential provided that half of the conformal curvature (i.e., self-dual or anti-self-dual) of the background vanishes. Thus, the Ricci-flat background is further restricted to be totally flat [18]. Our boundary three-geometry consists of one or two three-spheres, since these are the spatial sections occurring in the quantization of closed cosmological models.

As far as metric perturbations $h_{\mu\nu}$ are concerned, one would like to impose boundary conditions written in terms of projectors and/or first-order differential operators, completely invariant under infinitesimal diffeomorphisms, and leading to a self-adjoint second-order operator on $h_{\mu\nu}$. It has been proved in Ref. [12] that this may be obtained by working in the axial gauge (this being a *sufficient* condition to achieve self-adjointness) and hence setting all components of $h_{\mu\nu}$ equal to zero at the boundary.

For gravitino perturbations, one has a choice between non-local boundary conditions of the spectral type, and local boundary conditions motivated by local supersymmetry. In the former case, one sets to zero at the boundary half of ψ_μ^A and $\tilde{\psi}_\mu^{A'}$, i.e., those modes which multiply harmonics having positive eigenvalues of the intrinsic three-dimensional Dirac operator of the boundary. This is a non-local operation, in that it relies on a separation of the spectrum of such a three-dimensional elliptic operator into its positive and negative parts. Local boundary conditions for gravitinos involve instead complementary projectors,

but they are *not* completely invariant under infinitesimal diffeomorphisms [19]. Hence we focus on the choice of spectral boundary conditions in the axial gauge, and we refer the reader to Sec. IV of Ref. [20] for the analysis of local boundary conditions.

2. Simple supergravity in the axial gauge: one-loop results

In the Faddeev-Popov path integral for the semiclassical amplitudes of simple supergravity, one adds a gauge-averaging term to the original Euclidean action for gravitons and gravitinos, jointly with the corresponding ghost term [21]. In Ref. [12] it has been shown that, on imposing the boundary conditions

$$\left[h_{ij} \right]_{\partial M} = \left[h_{00} \right]_{\partial M} = \left[h_{0i} \right]_{\partial M} = 0 \quad , \quad (2.1)$$

in the axial gauge, one obtains a unique, smooth and analytic solution of the elliptic boundary-value problem, which picks out transverse-traceless perturbations. Thus, when flat Euclidean four-space is bounded by a three-sphere, the contribution of gravitons to the one-loop divergence is [22]

$$\zeta_{\text{TT}}(0) = -\frac{278}{45} \quad , \quad (2.2)$$

since gravitational ghost modes are forced to vanish everywhere in the axial gauge [12].

For gravitinos, denoting by ${}_{\epsilon}n_{CA'}$ the two-spinor version of the Euclidean normal to the boundary, the four-dimensional ghost operators in the axial gauge are found to be [20]

$$\mathcal{D}_C^A \equiv {}_{\epsilon}n_{CC'} \nabla^{AC'} \quad , \quad (2.3)$$

$$\mathcal{F}_{C'}^{A'} \equiv {}_{\epsilon}n_{CC'} \nabla^{CA'} \quad . \quad (2.4)$$

The corresponding ghost fields ν^C and $\mu^{C'}$ obey Neumann conditions at the three-sphere boundary of radius a . These result from requiring invariance of spectral boundary conditions for gravitino perturbations under infinitesimal gauge transformations. Hence one finds the discrete spectra $\lambda_n = (n+3)/a$, $\forall n \geq 0$, for \mathcal{D}_C^A , and $\tilde{\lambda}_n = n/a$, $\forall n \geq 0$, for $\mathcal{F}_{C'}^{A'}$. The $\zeta(0)$ value for ghosts is thus found to be [20]

$$\begin{aligned} \zeta(0) &= \zeta_H(-2, 3) - 3\zeta_H(-1, 3) + 2\zeta_H(0, 3) + 2 + \zeta_R(-2) + 3\zeta_R(-1) + 2\zeta_R(0) \\ &= -\frac{3}{4} + \frac{3}{4} = 0 \quad , \end{aligned} \quad (2.5)$$

where ζ_H and ζ_R are the Hurwitz and Riemann zeta-functions, respectively. Moreover, in the axial gauge, the four-dimensional elliptic operator acting on Rarita-Schwinger potentials is $\mathcal{O}^{\mu}_{\nu} \equiv -\square \delta^{\mu}_{\nu} + \frac{1}{\alpha} n^{\mu} n_{\nu}$. Covariant differentiation of the resulting eigenvalue

equation, and contraction with flat-space γ -matrices, jointly with spectral boundary conditions on ψ_μ^A and $\tilde{\psi}_\mu^{A'}$, imply that a unique solution exists, given by transverse-traceless perturbations. Their contribution to $\zeta(0)$ is [23,24]

$$\zeta_{\frac{3}{2}}(0) = -\frac{289}{360} \quad , \quad (2.6)$$

and hence the full $\zeta(0)$ value turns out to be [20]

$$\zeta(0)_{N=1 \text{ SUGRA}} = -\frac{278}{45} + \frac{289}{360} = -\frac{43}{8} \quad . \quad (2.7)$$

It should be emphasized that the cancellation of contributions of gauge and ghost modes in the axial gauge is a non-trivial property in the presence of boundaries, since no such cancellation occurs in covariant gauges. This has been proved by using both geometric formulae for heat-kernel asymptotics (corresponding to the Schwinger-DeWitt method) and explicit mode-by-mode calculations [9–11,15]. Moreover, even in the Coulomb gauge for Euclidean Maxwell theory such a cancellation was not found to occur in the presence of boundaries [14].

When two concentric three-sphere boundaries of radii τ_- and τ_+ occur, which is the case more relevant for quantum field theory, the equation obeyed by the gravitino eigenvalues by virtue of spectral boundary conditions turns out to be, for all $n \geq 0$ [20]

$$I_{n+2}(M\tau_+)K_{n+3}(M\tau_-) + I_{n+3}(M\tau_-)K_{n+2}(M\tau_+) = 0 \quad , \quad (2.8)$$

where I_n and K_n are the modified Bessel functions, and $\tau_+ > \tau_-$. This leads to [20]

$$\zeta_{\frac{3}{2}}(0) = 0 \quad , \quad (2.9)$$

and hence the full one-loop divergence is given by [20]

$$\zeta(0)_{N=1 \text{ SUGRA}} = -5 \quad , \quad (2.10)$$

since transverse-traceless graviton modes contribute -5 , and gravitino ghost modes vanish everywhere in this two-boundary problem [20].

3. Open problems

The results (2.7) and (2.10) seem to imply that simple supergravity is not even one-loop finite in the presence of boundaries, when the axial gauge and spectral boundary conditions are imposed. However, at least three outstanding problems remain. They are as follows.

(i) One has to understand whether the reduction to transverse-traceless perturbations for both gravitons and gravitinos is a peculiar property of the axial gauge only, within the framework of Faddeev-Popov formalism for quantum amplitudes. Indeed, such a reduction has not been found to occur in the case of local boundary conditions [20]. The relation between quantization schemes in different non-covariant gauges (e.g. axial vs. unitary) in the presence of boundaries deserves careful consideration.

(ii) Higher-N supergravity models are naturally formulated in backgrounds with a non-vanishing cosmological constant. How to choose the boundary three-geometry ? How to deal with antisymmetric tensor fields ? Is the one-loop divergence an unavoidable feature of any problem with boundaries ?

(iii) How to study higher-order effects in perturbation theory for simple or extended supergravity theories in the presence of boundaries.

Maybe a new age is in sight in the understanding of ultraviolet divergences in quantum supergravity. If this were the case, it would shed new light on perturbative properties in quantum gravity and quantized gauge theories.

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